Neutrinos in flat extra dimension: towards a realistic scenario

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ABSTRACT: We study the simple extension of the Standard Model in which an additional right handed neutrino propagates along a flat extra dimension, while the Standard Model fields are confined on a 3-brane. The fifth dimension is S^1/Z_2 orbifold compactified. The most generic Lagrangian (thus breaking the lepton number) is considered. In this scenario, the neutrino mass can be naturally suppressed. By studying systematically the fundamental parameter space, we show that the strong phenomenological constraints on mixing angles between active and sterile neutrinos (especially those derived from the SNO experiment data) do not conflict with the possibility of generating a realistic neutrino mass spectrum. As a second step, we explore the possibility of a successful leptogenesis through the decays of the Kaluza-Klein excitations of the right handed neutrino.

Contents

1. Introduction

In the recent days, our perspectives in the search for physics beyond the Standard Model (SM) have enriched considerably with the developments of theories with extra (compact) dimensions [1 – 3]. In particular, these extra spatial dimensions allow to lower the gravity scale down to the TeV [1, 2], thus addressing the long standing puzzle of the gauge hierarchy problem, when gravitational interactions feel these dimensions. Such TeV-scale Extra Dimensional (ED) models have attracted a great attention from the experimentalist community, since one can undertake their precision studies in the next generation colliders. Depending on the geometry of the extra dimensions (shape and/or size), and the field localization, a wide variety of extra dimensional models have been proposed. In addition to the gauge hierarchy problem, these models address several other questions in the different avenues of particle physics, cosmology and string theory.

Among a large number of possible realizations of the ED models, the scenario, with additional right handed neutrinos (being gauge singlet, and thus sterile) in flat compact extra dimensions and SM fields localized on a brane, provides an interesting alternative [4 – 7] capable of accounting for the observed light neutrino masses [8]. Also, the detailed experimental results on neutrino properties, in the context of this scenario, may be utilized to shed light on the bulk geometry (for phenomenological studies, see [4, 5, 9]).

In this paper we study this scenario based on the formalism of [11]. Our first aim is to determine whether a realistic neutrino spectrum can still be generated when the constraints on mixing angles between active and sterile neutrinos are taken into account, and particularly the severe constraint from the SNO experiment. Such constraints should indeed apply on the considered scenario, since the new right handed neutrino, as well as each component of its infinite massive tower of Kaluza-Klein (KK) excitations, constitute sterile neutrinos which mix with SM active neutrinos. We consider the general Lagrangian, which possess two distinct sources of Lepton number (L) breaking. The phenomenological constraints on neutrinos were applied, in [12], to the model with an L-conserving Lagrangian assuming no brane shifting.

Our second goal is to address the issue of a successful leptogenesis within such realistic ED model in agreement with experimental neutrino data. More precisely, by studying the decays of the heavy KK neutrinos, we explore the possibilities that these scenarios account for the observed baryonic asymmetry of the Universe by means of the Fukugita-Yanagida mechanism of leptogenesis [13]. In this mechanism an excess of L, generated by out-of-equilibrium L-violating decays of the heavy neutrinos (arising with the L-violating terms present in our Lagrangian), is converted into a Baryon number (B) asymmetry through $(B + L)$-violating sphaleron interactions. At this stage we mention a preliminary work [6] where the leptogenesis has also been considered in the same context of flat extra dimensions, but with an extended Higgs sector compared to the SM and without considering the various experimental neutrino data.

We will consider the case where gravity may propagate in an higher dimensional space of $[1 + (3 + n_g)]$ dimensions, where $n_g \geq 1$. Hence we have two variants of the scenario, one with $n_g = 1$ and the other with $n_g > 1$. In all cases, the SM particles are localized on a $[1 + 3]$ dimensional subspace (3-brane). For simplification reason, the sterile neutrinos are assumed to propagate in $[1 + (3 + 1)]$ dimensions.

The organization of the paper is as follows. We begin with a brief outline of the essential features of the extra dimensional scenario in section 2. The relevant phenomenological constraints on neutrinos are discussed in section 3, while our numerical results are presented and discussed in section 4. Next in section 5, the issue of leptogenesis is addressed. Lastly, we summarize our studies, and conclude, in section 6.

2. Minimal higher dimensional model for neutrinos

In this section we describe the minimal ED framework with a 5-dimensional iso-singlet right handed neutrino added to the field content of the SM [4, 11]. Here, we stick to one generation of SM neutrinos. The fifth flat dimension, along which propagates the right handed neutrino, is compactified over a S^1/Z_2 orbifold. The SM fields are localized on a 3-brane, whereas gravity propagates in the bulk.

The leptonic field content is,

$$L(x) \; = \; \begin{pmatrix} \nu_\ell(x) \\ \ell_L(x) \end{pmatrix}, \qquad \ell_R(x), \tag{2.1}$$

as in the SM, where ν_ℓ, ℓ_L, ℓ_R are 4-dimensional Weyl spinors, plus the 5-dimensional singlet neutrino,

$$N(x,y) = \begin{pmatrix} \xi(x,y) \\ \bar{\eta}(x,y) \end{pmatrix}, \tag{2.2}$$

where y parameterizes the compact fifth direction, and ξ, η are the 2-component spinors. The y-coordinate is compactified on a circle of radius R (the periodic boundary condition $N(x,y) = N(x, y + 2\pi R)$ is imposed) on the singlet neutrino field. Additionally, under the action of Z_2 symmetry, the two 2-component spinors may be associated with opposite parities,

$$\xi(x,y) = \xi(x,-y), \qquad \eta(x,y) = -\eta(x,-y). \tag{2.3}$$

The SM fields are localized at the brane, which, just for generality, we may assume to be at $y = a$, instead of at the orbifold fixed point $y = 0$. For this model, an effective Lagrangian respecting the 4-dimensional Lorentz invariance reads as,

$$\mathcal{L}_{\text{eff}} = \int\limits_0^{2\pi R} dy \left\{ \bar{N}\left(i\gamma^\mu \partial_\mu + \gamma_5 \partial_y \right) N - \frac{1}{2}\left(M N^T C^{(5)-1} \gamma_5 N + \text{h.c.} \right) \right.$$
$$\left. + \delta(y-a)\left[\frac{h_1}{(M_F)^{1/2}} L\tilde{\Phi}^* \xi + \frac{h_2}{(M_F)^{1/2}} L\tilde{\Phi}^* \eta + \text{h.c.} \right] + \delta(y-a)\mathcal{L}_{\text{SM}} \right\}, \tag{2.4}$$

where $\tilde{\Phi} = i\sigma_2 \Phi^*$ is the hypercharge-conjugate of the SM Higgs doublet Φ, with hypercharge $Y(\Phi) = 1$, $\mathcal{L}_{\text{SM}}$ is the SM Lagrangian, M is the Majorana mass for N (we do not specify its scale for the moment), $C^{(5)}$ is the 5-dimensional charge conjugation operator and M_F is the fundamental n_g-dimensional gravity scale, given by

$$M_P = (2\pi M_F R)^{n_g/2} M_F, \tag{2.5}$$

for the simple case where all the compactification radii are of equal size R, M_P being the 4-dimensional Planck scale. A Dirac mass term $m_D \bar{N} N$ is not allowed in eq. (2.4) because of the Z_2 discrete symmetry. Note that the Majorana mass term for N breaks the 5-dimensional Lorentz invariance as in [10].

Following eq. (2.3), the 2-component spinors ξ and η can be expanded in Fourier series as,

$$\xi(x,y) = \frac{1}{\sqrt{2\pi R}}\,\xi_0(x) + \frac{1}{\sqrt{\pi R}}\sum_{n=1}^\infty \xi_n(x)\,\cos\left(\frac{ny}{R}\right), \tag{2.6}$$

$$\eta(x,y) = \frac{1}{\sqrt{\pi R}}\sum_{n=1}^\infty \eta_n(x)\,\sin\left(\frac{ny}{R}\right), \tag{2.7}$$

where the chiral spinors $\xi_n(x)$ and $\eta_n(x)$ form an infinite tower of KK fields. Using these

expansions and integrating out the y-coordinate, the effective Lagrangian reduces to,

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \bar{\xi}_0(i\bar{\sigma}^\mu \partial_\mu)\xi_0 + \left(\bar{h}_1^{(0)} L\tilde{\Phi}^*\xi_0 - \frac{1}{2} M \xi_0\xi_0 + \text{h.c.} \right) \tag{2.8}$$

$$+ \sum_{n=1}^{\infty} \left[\bar{\xi}_n(i\bar{\sigma}^\mu \partial_\mu)\xi_n + \bar{\eta}_n(i\bar{\sigma}^\mu \partial_\mu)\eta_n + \frac{n}{R}\left(\xi_n\eta_n + \bar{\xi}_n\bar{\eta}_n \right) \right.$$

$$\left. - \frac{1}{2} M \left(\xi_n\xi_n + \bar{\eta}_n\bar{\eta}_n + \text{h.c.} \right) + \sqrt{2}\left(\bar{h}_1^{(n)} L\tilde{\Phi}^*\xi_n + \bar{h}_2^{(n)} L\tilde{\Phi}^*\eta_n + \text{h.c.} \right) \right]$$

in a basis in which M is positive, and with:

$$\bar{h}_1^{(n)} = \frac{h_1}{(2\pi M_F R)^{1/2}} \cos\left(\frac{na}{R} \right) = \left(\frac{M_F}{M_P} \right)^{1/n_g} h_1 \cos\left(\frac{na}{R} \right) = \bar{h}_1 \cos\left(\frac{na}{R} \right), \tag{2.9}$$

$$\bar{h}_2^{(n)} = \frac{h_2}{(2\pi M_F R)^{1/2}} \sin\left(\frac{na}{R} \right) = \left(\frac{M_F}{M_P} \right)^{1/n_g} h_2 \sin\left(\frac{na}{R} \right) = \bar{h}_2 \sin\left(\frac{na}{R} \right). \tag{2.10}$$

For deriving the last two equalities on the right hand sides of eqs. (2.9)–(2.10), we have made use of eq. (2.5).

Eqs. (2.9) and eq. (2.10) tell us that the reduced 4-dimensional Yukawa couplings $\bar{h}_{1,2}^{(n)}$ can be suppressed by many orders depending on the hierarchy between M_P and M_F; for example, if gravity and the bulk neutrino feel the same number of extra dimensions, say $n_g = 1$, then these couplings are suppressed by a factor $M_F/M_P \sim 10^{-15}$, for $M_F \approx 10\,\text{TeV}$ (see also [4, 5]).

From eq. (2.2) we observe that ξ and $\bar{\eta}$ have the same lepton number. Thus, the simultaneous presence of the two operators $L\tilde{\Phi}^*\xi$ and $L\tilde{\Phi}^*\eta$ in eq. (2.8) leads to lepton number violation. If the brane were located at one of the two orbifold fixed points ($y = 0$ or πR), the operator $L\tilde{\Phi}^*\eta$ would have been absent in eq. (2.8) as a consequence of the discrete Z_2 symmetry. The two operators can coexist, leading to breaking of the lepton number, if we allow the brane to be shifted by an amount $a(\neq 0)$ from the orbifold fixed points. In fact, it is possible to perform such a shifting of the brane (even in a continuous way) respecting the Z_2 invariance of the original higher dimensional Lagrangian under certain restrictions in Type-I string theories [14]. As indicated in [4, 11], the Z_2 invariance can be taken care of by allowing the replacements,

$$\xi\,\delta(y-a) \to \frac{1}{2}\,\xi\left[\delta(y-a) + \delta(y+a-2\pi R) \right],$$

$$\eta\,\delta(y-a) \to \frac{1}{2}\,\eta\left[\delta(y-a) - \delta(y+a-2\pi R) \right], \tag{2.11}$$

with $0 \leq a < \pi R$ and $0 \leq y \leq 2\pi R$. Obviously, a Z_2-invariant implementation of brane-shifted couplings would require the existence of at least two branes placed at $y = a$ and $y = 2\pi R - a$. A remarkable feature of the brane-shifted framework was pointed out [11], where it has been shown that in such a framework it is possible to completely de-correlate the effective Majorana-neutrino mass $\langle m \rangle$, that is effectively measured, and the scale of light neutrino masses, as to have $\langle m \rangle$ within an observable range.

Therefore, the Lagrangian (2.8) contains two types of Majorana neutrino mass term (involving respectively the parameters M and $\bar{h}_2^{(n)}$) which lead both to a breaking of L. This L-breaking is a necessary ingredient of leptogenesis.

Following the notations of reference [4], we now introduce the weak basis for the KK Weyl-spinors,

$$\chi_{\pm n} = \frac{1}{\sqrt{2}}(\xi_n \pm \eta_n),\tag{2.12}$$

followed by a rearrangement of ξ_0 and $\chi_n^\pm$ states, such that, for a given value of k (say, $k = k_0$) the smallest diagonal entry of the neutrino mass matrix is

$$|\varepsilon| = \min\left(\left|M - \frac{k_0}{R}\right|\right) \leq 1/(2R).\tag{2.13}$$

In this new basis the reordered 4-component Majorana-spinor vector Ψ_ν is the following,

$$\Psi_\nu^T = \left[\begin{pmatrix}\chi_{\nu_\ell} \\ \bar{\chi}_{\nu_\ell}\end{pmatrix}, \begin{pmatrix}\chi_{k_0} \\ \bar{\chi}_{k_0}\end{pmatrix}, \begin{pmatrix}\chi_{k_0+1} \\ \bar{\chi}_{k_0+1}\end{pmatrix}, \begin{pmatrix}\chi_{k_0-1} \\ \bar{\chi}_{k_0-1}\end{pmatrix}, \cdots, \begin{pmatrix}\chi_{k_0+n} \\ \bar{\chi}_{k_0+n}\end{pmatrix}, \begin{pmatrix}\chi_{k_0-n} \\ \bar{\chi}_{k_0-n}\end{pmatrix}, \cdots\right]\tag{2.14}$$

while the effective Lagrangian for right handed neutrinos reduces to,

$$\mathcal{L}_{\text{kin}} = \frac{1}{2}\bar{\Psi}_\nu\left(i\,\slashed{\partial} - \mathcal{M}_\nu^{\text{KK}}\right)\Psi_\nu,\tag{2.15}$$

where $\mathcal{M}_\nu^{\text{KK}}$ is the corresponding neutrino mass matrix given by,

$$\mathcal{M}_\nu^{\text{KK}} = \begin{pmatrix} 0 & m^{(0)} & m^{(-1)} & m^{(1)} & m^{(-2)} & m^{(2)} & \cdots \\ m^{(0)} & \varepsilon & 0 & 0 & 0 & 0 & \cdots \\ m^{(-1)} & 0 & \varepsilon - \frac{1}{R} & 0 & 0 & 0 & \cdots \\ m^{(1)} & 0 & 0 & \varepsilon + \frac{1}{R} & 0 & 0 & \cdots \\ m^{(-2)} & 0 & 0 & 0 & \varepsilon - \frac{2}{R} & 0 & \cdots \\ m^{(2)} & 0 & 0 & 0 & 0 & \varepsilon + \frac{2}{R} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}.\tag{2.16}$$

The most important consequence of such a rearrangement is that the mass scale M, which we did not specify earlier but which could be as large as possible, is now replaced by the light mass scale ε. The mass entries in the first row and first column of matrix (2.16) are given by the relation,

$$m^{(n)} = \frac{v}{\sqrt{2}}\left[\bar{h}_1\cos\left(\frac{(n-k_0)a}{R}\right) + \bar{h}_2\sin\left(\frac{(n-k_0)a}{R}\right)\right] = m\cos\left(\frac{na}{R} - \phi_h\right),\tag{2.17}$$

with,

$$m = \frac{v}{2}\sqrt{\frac{h_1^2 + h_2^2}{\pi M_F R}},\tag{2.18}$$

$$\phi_h = \tan^{-1}\left(\frac{h_2}{h_1}\right) + k_0\frac{a}{R},\tag{2.19}$$

where v is the vacuum expectation value of the SM Higgs boson.

We observe that the structure of neutrino mass matrix (2.16), originating from a combination of Majorana and Dirac mass terms, is different from the pure Dirac mass case considered in the preliminary work [12].

3. Constraints for a realistic spectrum

The neutrino spectra is obtained by diagonalizing the mass matrix $\mathcal{M}_\nu^{\mathrm{KK}}$ of eq. (2.16). It has been shown in [11], for a one-generation case, that the eigenvalue equation for a restricted class of cases with $a = \pi R/q$, where q is an integer greater than 1, is easily tractable analytically. We have, in the generic case, solved the characteristic eigenvalue equation numerically to obtain the eigenvalues of $\mathcal{M}_\nu^{\mathrm{KK}}$, sticking to the one-generation case. We have diagonalized the $N \times N$ matrix of eq. (2.16) for a N satisfying $M_F > M_{\max}$ (see end of the paragraph). We have systematically checked that the N value used (typically ~ 500 for the smallest R^{-1} values to ~ 10 for the largest R^{-1} values) constitutes a dimension at which the lightest eigenvalues converge and are completely stabilized (with increasing N). We have performed a scan over the whole parameter space of the ED model described above in order to find the regions in agreement with basic experimental constraints on neutrinos. The total parameter space consists of R^{-1}, ε, the complex Yukawa couplings h_1 and h_2, the brane-shift parameter a, and the phase ϕ_h defined in eq. (2.19). From eq. (2.5), the effective cut off scale M_F for a given R^{-1} is completely fixed for $n_g = 1$; while for $n_g > 1$, it depends on n_g and the respective compactification radii (respecting $M_F > M_{\max}$, where $M_{\max}$ is the mass of the heaviest neutrino eigenstate considered).

While scanning over the total parameter space described above in order to search for a realistic neutrino mass spectrum, we have to ensure several constraints described in the following;

- There exist phenomenological constraints on the mixing angles between active neutrinos and sterile ones. These constraints apply on the present model, as the 0-mode and KK excitations of the additional right handed neutrino behave exactly as sterile neutrinos $-$ the right handed neutrino having no electroweak interactions (see e.g. [15]).

  For instance, in a model with a unique sterile neutrino, and assuming for simplicity one lepton flavor, the mixing angle θ_s between the active and sterile neutrino is constrained typically by $\tan \theta_s^2 \lesssim 10^{-6} - 10^{-1}$ for $\Delta m^2 \in [10^{-12} - 10^2]$ eV^2 (Δm^2 being the mass eigenvalue squared difference) [16] from cosmological and astrophysical considerations combined with data from atmospheric, solar (including SNO), reactor and short base-line experiments. In the limiting case $\Delta m^2 \gg \Delta m_{\mathrm{sun}}^2 \sim 10^{-5}$ eV^2, the SNO bound on the fraction of oscillating sterile neutrinos coming from the sun $\eta_s = \sin^2 \theta_s / 2$ [16], becomes approximatively [17, 18]

$$\eta_s \lesssim 1.2 \times 10^{-1} \quad \text{at } 1\sigma. \tag{3.1}$$

  Similarly, in our framework, one can also take as an approximation that the χ_{k_0} state, whose mass ε is the first appearing as a diagonal matrix element in eq. (2.16),

is the only sterile neutrino that may not decouple from the active SM neutrino ν_ℓ. Indeed, by definition of ε in eq. (2.13), the following $\chi_{k_0 \pm n}$ states get larger and larger diagonal mass matrix elements $\varepsilon \pm n/R$ and thus decouple more and more from ν_ℓ, given the symmetric structure of the mass matrix in eq. (2.16). Therefore, under the above approximation, the SNO bound on θ_s apply in our framework also, where the mixing angle θ_s is deduced only from the 2×2 block matrix of eq. (2.16):

$$\tan 2\theta_s \approx \frac{2m^{(0)}}{\varepsilon}. \tag{3.2}$$

The above SNO bounds impose a significant upper limit on θ_s leading to a case comparable to the see-saw mechanism [19] (see also the third point) so that the smallest eigenvalue m_{ν_1} and the next-to-smallest eigenvalue m_{ν_2} read as

$$m_{\nu_1} \approx \frac{m^{(0)}}{\varepsilon} m^{(0)}, \quad m_{\nu_2} \approx \varepsilon. \tag{3.3}$$

The constraint in eq. (3.1) imposes $m^{(0)} \ll \varepsilon$ (c.f. eq. (3.2)), which may be achieved in the following two cases (see eq. (2.17)):

$$m \ll \varepsilon, \quad \text{and/or} \quad \phi_h \simeq \pi/2 + 2q\pi, \tag{3.4}$$

where q is an integer.

The previous approximations regarding the implications of the constraints on sterile neutrino mixing will turn out to be useful in the interpretation of our results. In order to quantitatively take into account the effects of the KK tower of neutrino states when applying the main SNO constraint, we have imposed the bound: $P(\nu_e \to \nu_s) < 0.40$ at 90% C.L. (coming from the SNO data [20] combined with the Super-Kamiokande results [21]). $P(\nu_e \to \nu_s)$ represents the transition probability between solar SM electron-neutrino ν_e and sterile neutrino ν_s, and, is given by $P(\nu_e \to \nu_s) = 1 - \sum_{\ell=e,\mu,\tau} P(\nu_e \to \nu_\ell)$ where $P(\nu_e \to \nu_\ell) = \sum_n |V_{en}|^2 |V_{\ell n}|^2$ [22] with $\ell = e$ in the one active flavor case. The sum here is taken over the index n, which labels the neutrino mass eigenstates, and, V diagonalizes the neutrino mass matrix eq. (2.16). The sterile KK excitations are little affected by matter effects [12]. Following the same numerical approach for N as described in the beginning of this section, we have checked that for the chosen value of N (corresponding to the maximum value of n truncating the sum involved in the $P(\nu_e \to \nu_\ell)$ expression), $P(\nu_e \to \nu_s)$ reaches a value which is not significantly sensitive to further increases of the mass matrix dimension. In other terms, a satisfactory level of convergence can be reached in the summation of KK excitations. In fact, the stabilized values of $P(\nu_e \to \nu_s)$ for our case turn out to be systematically several orders of magnitude smaller than the combined SNO-Super-Kamiokande bound mentioned previously, in the final regions of parameter space which respect all the constraints discussed in this section.

- Due to the see-saw structure of the mass matrix, the two eigenstates ν_1 and ν_2 are respectively composed mainly by the SM neutrino ν_ℓ and sterile state χ_{k_0}. Hence

one can apply the phenomenological bound on m_{ν_1} derived for SM neutrinos, which in the case of one flavor is typically [23]

$$m_{\nu_1} \sim 10^{(-2 \ , \ 0)} \text{ eV}. \tag{3.5}$$

- We have also ensured that the mass of the next-to-lightest[1] eigenstate ν_2 is larger than the mass of the Higgs boson, typically

$$m_{\nu_2} \geq 200 \text{ GeV}, \tag{3.6}$$

for our purpose which deals with its possible lepton-number violating decay into Higgs and leptons: $\nu_2 \rightarrow \ell^\pm \phi^\mp$ (the three-body decay channel of the ν_2 is $\nu_2 \rightarrow \ell^\pm \ell^\mp \nu_1$).

The choice of this condition $m_{\nu_2} > 200$ GeV is important for our study of leptogenesis (in section 5). We could also have imposed this constraint on any higher-mass neutrino eigenstate $m_{\nu_i} > 200$ GeV with $i > 2$, and studied the decay $\nu_i \rightarrow \ell^\pm \phi^\mp$. However, any higher state ν_i $(i > 2)$, in addition to its decay channel $\nu_i \rightarrow \nu_1 \ell^+ \ell^-$, possesses several other decay channels: their decays into lighter eigenstates $\nu_i \rightarrow \nu_j \ell^+ \ell^-$ or $\nu_i \rightarrow \nu_j \nu_1 \nu_1$ with $i > j \geq 2$. How the presence of these additional decay channels is going to affect the Boltzmann equations and the washout factors, and thus the resulting lepton abundance, is a matter of non-trivial calculation that we will not address here. Such additional decay channels do not exist for the state ν_2, as the decays $\nu_2 \rightarrow \nu_2 \nu_1 \nu_1$ and $\nu_2 \rightarrow \nu_2 \ell^+ \ell^-$ are not possible kinematically. Here we consider the possibility of the ED model to yield a successful leptogenesis via the ν_2 decay.
With this bound $m_{\nu_2} \geq 200$ GeV we are in the case where $\Delta m^2 = m_{\nu_2}^2 - m_{\nu_1}^2 \gg \Delta m_{\text{sun}}^2$ so that the relevant SNO bound is the one given in eq. (3.1). Furthermore, this condition forces m_{ν_2} to be much larger than m_{ν_1} and so constitutes another justification for approximating the two lightest eigenvalues by the see-saw formulas (3.3).

At this stage the following remarks on neutrino mass are in order. There are typically three different mass suppression mechanisms in the extra dimensional scenario, that may suppress the mass of the SM neutrino.

(i) The usual see-saw like suppression, from a heavy Majorana mass M for the right handed neutrino.

(ii) An effective ED see-saw type mechanism resulting from the massive KK tower [4, 6], where the KK excitations of the additional right handed neutrino play the rôle of the right handed neutrino in the usual see-saw mechanism.

(iii) A suppression of the Yukawa couplings from the ED wavefunction overlap mechanism affecting the neutrino mass via the ED volume factor $(M_F R)^{1/2}$ (see eq. (2.18)) and the factor $\cos(na/R - \phi_h)$ (see eq. (2.17)).

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[1] In our notations we order the mass eigenvalues increasingly, i.e. $m_{\nu_1} \leq m_{\nu_2} \leq \cdots$.

The main motivation for working in an ED scenario is to generate small SM neutrino masses using the second and third mechanisms only, i.e. the usual see-saw mechanism does not constitute the main suppression origin. Is that possible? As we have already discussed, the diagonal matrix element ε contributes dominantly to the eigenvalue m_{ν_1} since it is the lightest one. Hence in order to insure that the neutrino mass suppression is not mainly due to the usual see-saw mechanism, one can impose the condition $M \neq \varepsilon$, which is equivalent to $M > 1/2R$ (see upper left 2×2 block matrix in eq. (2.16)). This condition can always be satisfied since the mass scale M never appears in this scenario directly: the presence of the infinite tower of KK states guarantees that only the value of M modulo R^{-1}, namely ε (in absolute values) plays a physical rôle [4, 11].

We now discuss the ranges considered for each fundamental free parameter. First we made sure that it is mainly the second and/or the third mechanism (described previously) that suppresses m_{ν_1}, by letting the numerical values of the Yukawa couplings h_1 and h_2 to be within the natural range $\sim 0.1 - 5$ with the respective phases ranging between 0 and π. The phase ϕ_h was also varied within 0 and π. We varied R^{-1} from a few tens of eV up to $\sim 10^{19}\,\text{GeV}$. For $R^{-1} \gtrsim 10^{19}\,\text{GeV}$, the heaviest state considered has a mass $M_{\max}$ greater than M_F, i.e. our low energy effective theory looses its predictivity. The range of R^{-1} being so huge, we considered a logarithmic binning of the scale of R^{-1}. For each R^{-1}, the parameter ε may also vary within a large range, especially when R^{-1} is large, following eq. (2.13). So we also scan over ε by choosing randomly the values of α in $\varepsilon = 10^\alpha$. The brane shift parameter a was scanned in a similar way in $0 < a < \pi R$. Finally, we performed a scan by *randomly* choosing $\sim 10^6$ points, so that each point corresponds to a *distinct but random* combination of all the parameters.

4. Results and discussion

When scanning over the parameter space $\{R^{-1}, \varepsilon, \phi_h, h_1, h_2, a\}$ we have imposed a realistic spectra, i.e. one respecting all the constraints mentioned in the previous section, by using the numerical estimation of this neutrino mass spectra. We have explored the two typical situations allowing for such realistic spectra (*c.f.* eq. (3.4)) within two scenarios characterized by $n_g = 1$ and $n_g > 1$. Our observations and discussion for both the scenarios are given below. We have also included, in each case, possible analytical interpretations of our results.

4.1 Scenario with $n_g = 1$

(i) **The scale** M_F**:** We recall (see eq. (2.5)) that the cut off scale M_F depends only on R^{-1} for $n_g = 1$. As we increase the scale of R^{-1} from a few tens of eV to $\sim 10^{19}\,\text{GeV}$, the corresponding scale M_F increases from $\sim 10^9\,\text{GeV}$ to $\sim 10^{19}\,\text{GeV}$, following eq. (2.5). This makes the scale m in eq. (2.18) increasing from $\sim 10^2\,\text{eV}$ to $\sim 10^3\,\text{GeV}$.

(ii) **The parameter** R^{-1}**:** We observed that the energy scale R^{-1} has to be $\gtrsim 400\,\text{GeV}$ (corresponding to $M_F \gtrsim 10^{13}\,\text{GeV}$) for generating a realistic neutrino spectrum (see

figure 1). The reason is that $R^{-1} \geq 2|\epsilon|$ (from eq. (2.13)) with $\epsilon \approx m_{\nu_2}$ from eq. (3.3) and we have imposed $m_{\nu_2} \geq 200\,\mathrm{GeV}$. With regard to the neutrino mass, what is the physical meaning of this lower bound on R^{-1}? We know from eq. (2.5), that as R^{-1} increases, M_F also increases but at a much slower rate compared to R^{-1}. As a result, the quantity $1/\sqrt{M_F R}$ increases with R^{-1} so that the volume suppression mechanism for Yukawa couplings becomes less and less effective (see eqs. (2.9), (2.10)), for a fixed $\cos\phi_h$. Besides, the hierarchy between $m^{(0)} \propto 1/\sqrt{M_F R}$ and R^{-1} increases with R^{-1}, so that the effective ED see-saw mechanism becomes more effective in the reduction of m_{ν_1} if ε is set at its maximum value $1/2R$. Indeed the denominator of m_{ν_1} (eq. (3.3)) clearly increases at a faster rate compared to the numerator $m^{(0)}$ - thus making the see-saw formulae more effective as R^{-1} increases. Therefore, the fact that there is a minimum value for R^{-1} means that the see-saw mechanism plays at least a certain rôle in the neutrino mass suppression.

(iii) **The parameter ε:** For all $R^{-1} \gtrsim 400\,\mathrm{GeV}$, there exists a minimum value of ε, starting from which we end up with a realistic spectrum for all possible higher values of ε ($\leq 1/2R$). This minimum value is significantly larger than $200\,\mathrm{GeV}$ (recall that $\varepsilon \approx m_{\nu_2} \geq 200\,\mathrm{GeV}$) for $1/R \gtrsim 10^8\,\mathrm{GeV}$, as we see on figure 1 which shows the plot of ε versus $1/R$. This can be understood as follows.

This figure results from the general scan, and thus includes both the cases $m \ll \varepsilon$ and $\phi_h \sim \pi/2 + 2q\pi$. The three straight lines correspond to the ε values obtained analytically from the approximate see-saw relation in eq. (3.3), for $m_{\nu_1} = 1\,\mathrm{eV}$, $0.1\,\mathrm{eV}$ and $0.01\,\mathrm{eV}$, and $\sqrt{h_1^2 + h_2^2} = 4$ and $\phi_h = 0$ (i.e. $\cos\phi_h = 1$). Therefore, the points obtained close to these lines correspond to the case where the m_{ν_1} suppression is mainly due to the configuration (1). The regions far below these lines indicate the simultaneous occurrence of both configuration (1) and (2) i.e. $m \ll \varepsilon$ as well as $\phi_h \sim \pi/2 + 2q\pi$. Starting from these regions and approaching higher R^{-1} values, m increases, making the m_{ν_1} suppression from (1) less effective. Thus if we are to generate the same amount of suppression in m_{ν_1} (or equivalently the same smallness of ratio $m^{(0)}/\varepsilon$) for a fixed ε, the m_{ν_1} suppression coming from case (2) must be more effective, i.e. ϕ_h has to be closer to $\pi/2$. This condition on ϕ_h basically reduce the allowed range for ϕ_h, so that the scan misses the allowed ϕ_h values. This explains the large vacant area in figure 1 below the three lines. Therefore, the easier way to generate the suppression in the higher R^{-1} region is by case (1), i.e. by pushing the minimum value of ε to be still higher to account for the increase in m. Quite naturally therefore, the available parameter region for higher R^{-1} decreases, and is concentrated near maximal values of ε. Nevertheless, because of choosing to scan over the ϕ_h angle, the particular regions where $ak_0 \ll R$ and $h_1 \ll h_2$ (leading to a ϕ_h value close to $\pi/2$) have been missed. Anyway such a hierarchy between the Yukawa couplings h_1, h_2 would have no theoretical explanation and, in contrast, the interest of the considered ED model is to have $h_1 \sim h_2 \sim 1$ while suppressing the neutrino masses mainly from the second and/or third mechanism described above (the purely ED mechanisms).
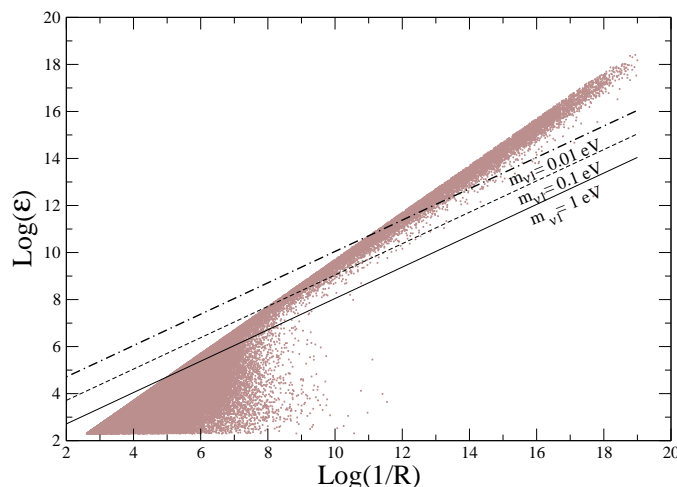
Figure 1: Plot of ε (in GeV) versus $1/R$ (in GeV) obtained from the scan. The three straight lines represent the analytical approximation of eq. (3.3) for the indicated values of m_{ν_1}.

One must realize that the specific points in figure 1, which are such that $\varepsilon \ll 1/R$, correspond to a value of M extremely close to k_0/R (see eq. (2.13)) and in turn to a situation of fine-tuning between the three parameters M, k_0 and R.

(iv) **The parameters ϕ_h and a:** First let us discuss the spectrum. Since we are always working in a parameter region such that the off-diagonal elements of the mass matrix of eq. (2.16) are much smaller than the diagonal elements, the eigenvalues m_{ν_i} of the matrix, up to a good approximation (see the section A), are equal to the diagonal elements for $i \geq 2$: $m_{\nu_2} \sim \varepsilon$, $m_{\nu_3} \sim \varepsilon - 1/R$, $m_{\nu_4} \sim \varepsilon + 1/R$, and so on. In general, the masses of the higher neutrino states can be expressed as $m_{\nu_{(2p+1)}} \sim \varepsilon - p/R$, $m_{\nu_{(2p+2)}} \sim \varepsilon + p/R$, where p is an integer ≥ 1. This clearly tells us that for a given R^{-1}, the nature of the resulting spectra is *primarily* dictated by the choice of ε. For example, if $\varepsilon = 1/2R$, the resulting spectrum becomes almost pairwise degenerate with $|m_{\nu_2}| \sim |m_{\nu_3}|$ (see eq. (3.3)), $|m_{\nu_4}| \sim |m_{\nu_5}|$, and so on, while, for all the other choices of ε, the resulting spectrum is hierarchical with $m_{\nu_2} < m_{\nu_3} < m_{\nu_4} < m_{\nu_5} < \cdots$

In particular, we found that for a given R^{-1}, the overall nature of the resulting spectrum *feebly* depends on the choice of the parameters ϕ_h and a. This is expected since these parameters appear *only* in the non-diagonal mass-matrix elements which are much smaller than the diagonal ones for any ϕ_h and a values (as those enter via arguments of a cosine function). We have found that for any $R^{-1} \gtrsim 400\,\mathrm{GeV}$, if $\varepsilon \sim 1/2R$, then *all* possible combinations of ϕ_h and a yield realistic spectra. Naturally, these possible combinations of ϕ_h and a include $a = 0$ as well. For possible lower values of ε, which may not correspond to $m \ll \varepsilon$, a realistic spectra requires the
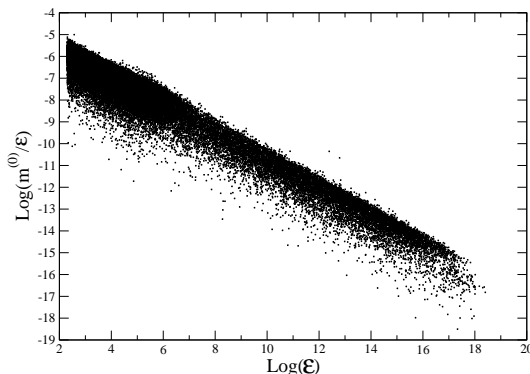
Figure 2: Plot of $m^{(0)}/\varepsilon$ versus ε (in GeV).

second scenario $\phi_h \sim \pi/2 + 2q\pi$ (as already discussed). Note that for this case also, $a = 0$ is a natural possibility. Thus the important conclusion that we made from these observations was that a non-zero brane-shift is *not essential* for a realistic mass spectrum.

Let us compare our observations with those of ref. [11]. The authors of reference [11] found that when $\varepsilon = 0$, $\phi_h = 0$ and $a = 0$, the mass spectra consists of massive pairwise degenerate neutrino states without any massless state. They also found that when $\varepsilon = 1/2R$, $\phi_h = 0$ and $a = 0$, there is an almost massless state, and the remaining spectra consists of pairwise degenerate massive neutrino states. We have found the same observation for these two points.

They further found that when $\varepsilon \neq 0$ and $\neq 1/2R$, $\phi_h = 0$ and $a = 0$, there is no massless state and the mass spectra is hierarchical. We indeed find an *almost massless* neutrino state in this case too, the overall spectrum being hierarchical.

They also found that *unless* $\varepsilon = 1/2R$, $\phi_h = \pi/4$ and $a = \pi R/2$, the mass spectrum consists of massive non-degenerate KK neutrinos. We find indeed that for $\varepsilon = 1/2R$, $\phi_h = \pi/4$ and $a = \pi R/2$, the high neutrinos states are pairwise degenerate.

However, we found that for several other combinations of these three parameters one can also get a realistic mass spectrum. For example, we mentioned before that for $\varepsilon = 1/2R$, we always get the spectra consisting of an almost massless state and other states pairwise degenerate, irrespective of the numerical values of ϕ_h and a. Our larger allowed domain of parameter space is due to the additional de-correlation condition required in [11], and not considered here. In particular, a non-zero brane-shift parameter is found to be necessary in [11] in order to account for this complete de-correlation of the effective Majorana mass term $\langle m \rangle$ and the scale of light neutrino mass (the motivation being to have $\langle m \rangle$ in a potentially observable range). In contrast, we find the brane-shift to be not essential for a realistic mass spectrum.

(v) **The quantity** $m^{(0)}$**:** We finish this section by determining which of the two possible ED mechanisms responsible for the suppression of m_{ν_1} (discussed in Sec 3) plays the dominant rôle (for different values of R^{-1}). Our discussion is based on the plot of $m^{(0)}/\varepsilon$ versus ε in figure 2. One first observes clearly from the figure that systematically $m^{(0)} \leq 10^{-5}\varepsilon$ which means that the effective ED see-saw mechanism plays *always* a significant rôle in reducing m_{ν_1} (see eq. (3.3)), as we have already noticed. One see typically that the m_{ν_1} suppression, relative to the electroweak symmetry breaking scale ($v \sim 10^2\,\text{GeV}$), can be due at 50% from the effective ED see-saw mechanism and at 50% from the higher dimensional mechanism based on wave function overlap, a situation corresponding to $\varepsilon \sim 10^2\,\text{GeV}$ and $m^{(0)} \sim 10^{-3}\,\text{GeV}$ (such that $m^{(0)}/v \sim m^{(0)}/\varepsilon \sim 10^{-5}$). The suppression can even come *purely* from the effective ED see-saw mechanism, namely $m^{(0)}/\varepsilon \sim 10^{-11}$ with $m^{(0)} \sim v \sim 10^2\,\text{GeV}$. Thus the fact that $m^{(0)}/\varepsilon \lesssim 10^{-5}$ means that the ED see-saw mechanism is systematically the dominant one.

This can interpreted in the following terms. All the conditions that we have imposed resulted in a lower bound on R^{-1} (see above).[2] Now, as R^{-1} increases, the extra compact bulk space decreases, so that the wavefunction overlap factor increases, making the suppression of m_{ν_1} from the geometrical mechanism less effective. Hence, the different constraints (including the SNO one) limit the wave function overlap mechanism effect. Nevertheless, thanks to the ED see-saw mechanism, a realistic spectrum can still be generated.

4.2 Results for the $n_g > 1$ scenario

We consider now another scenario where the right handed neutrino propagates only along one extra dimension whereas gravity propagates in the whole bulk, the number n_g and sizes of extra dimensions being adjusted (with eq. (2.16) when all of them have the same size $2\pi R$) such that M_F is as low as $\sim \mathcal{O}(1)\,\text{TeV}$ in order to address the gauge hierarchy problem. Recall that the solution to the gauge hierarchy problem within the ADD model for $n_g = 1$ is excluded by experimental arguments relative to the Newton law modification.[3]

The radius, as found previously, is bounded from above by $R^{-1} \gtrsim 400\,\text{GeV}$, since $R^{-1} \geq 2\,|\varepsilon|$, $\varepsilon \approx m_{\nu_2}$ and $m_{\nu_2} \geq 200\,\text{GeV}$. As a consequence, $m^{(0)} \sim v/\sqrt{M_F R} \gtrsim 10^2\,\text{GeV}$[4] as $M_F \sim \mathcal{O}(1)\,\text{TeV}$, so that there is no significant suppression in $m^{(0)}$ (and hence in m_{ν_1}), relative to the electroweak scale v, coming from the ED overlap mechanism. Thus the neutrino mass suppression must originate principally from the ED see-saw mechanism. For that purpose, one should have $m^{(0)}/\varepsilon \sim 10^{-11}$ (see eq. (3.3)) which together with $m^{(0)} \sim v/\sqrt{M_F R}$ and $|\varepsilon| < 1/2R$ leads to $R^{-1} \gtrsim 10^{26}\,\text{GeV}$: an amount much larger

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[2]In particular, the SNO bound constrains the mixing angles between active and sterile neutrinos, by this way forcing typically the KK masses ($\propto 1/R$) to increase.

[3]In our case for the $n_g = 1$ scenario, the cutoff scale was set around $10^{13}\,\text{GeV}$ and more. This model, for sure, did not address the gauge hierarchy issue.

[4]Unless of course $\phi_h \sim \pi/2 + 2q\pi$, but in order to decrease significantly the order of magnitude of this lower bound on $m_{(0)}$, ϕ_h should be extremely close to $\pi/2 + 2q\pi$ which has a priori no theoretical justification.

than the fundamental scale $M_F \sim \mathcal{O}(1)\,\mathrm{TeV}$. Thus, for the $n_g > 1$ scenario, generating a realistic neutrino spectrum and addressing the gauge hierarchy issue simultaneously, is only possible if there exists a compactification scale $1/R$ extremely different from the fundamental scale M_F (hence another hierarchy has to be introduced). Besides, such a scenario is automatically in a non-predictable regime, since the condition $M_{\max} < M_F$ translates into $1/R \lesssim \mathrm{TeV}$.

5. Leptogenesis

One interesting question to address is the possibility of having successful leptogenesis in these ED scenarios. As has been argued in [6], the infinite KK tower of neutrinos associated with the 5-dimensional sterile neutrino may act as an infinite series of CP-violating resonators. Thus, we probed the ED scenarios to see whether they can generate sufficient lepton asymmetry.

5.1 Additional constraints and expectations

In the previous section we saw that the lightest neutrino ν_1 in the generated spectrum fulfills the absolute scale bound. We also constrained the next to lightest neutrino ν_2 to be heavier than the Higgs mass, so that it may decay at a temperature corresponding to its mass and produce lepton asymmetry [13]. The out-of equilibrium condition may be written as the following condition on the decay width of ν_2,

$$\Gamma_{\nu_2} \simeq \frac{K_{22} m_{\nu_2}}{8\pi} < H(T = m_{\nu_2}), \quad K_{jj} \equiv [\lambda^\dagger \lambda]_{jj}, \tag{5.1}$$

where H is the Hubble expansion rate at $T = m_{\nu_2}$, and λ_j is the modified Yukawa coupling for the neutrino mass eigenstate ν_j. The initial Yukawa couplings, λ', can be read from eq. (2.17): $m^{(j)} = \lambda'_j v / \sqrt{2}$. When the complex symmetric mass matrix in eq. (2.16) is diagonalized, the weak states Ψ_ν of eq. (2.14) transform to the mass eigenstates Ψ_ν^{mass} following $\Psi_\nu = V \Psi_\nu^{\mathrm{mass}}$, where V is the diagonalizing matrix. Thus the initial Yukawa couplings for right handed neutrinos λ'_i also transform into the modified Yukawa couplings λ_j via the V matrix. We note the Yukawa couplings λ_i instead of λ_{i1}, where the index 1 is fixed as it corresponds to the unique left handed SM lepton (we remind that we work under the simplification assumption of one family) which moreover does not develop a KK tower. The condition in eq. (5.1) is the first filter to ensure a successful thermal leptogenesis. The other necessary condition is to generate enough CP asymmetry, the asymmetry being defined by,

$$\epsilon_{\mathrm{CP}} = \frac{\Gamma(\nu_2 \to \ell\phi) - \Gamma(\nu_2 \to \bar{\ell}\bar{\phi})}{\Gamma(\nu_2 \to \ell\phi) + \Gamma(\nu_2 \to \bar{\ell}\bar{\phi})}. \tag{5.2}$$

This asymmetry gets a non-zero value due to the interference between the tree level diagram of figure 3a and one-loop diagrams of figure 3b and 3c. The asymmetry is given by,

$$\epsilon_{\mathrm{CP}} = \frac{1}{8\pi K_{22}} \sum_{j \neq 2} \mathrm{Im}[K_{2j}^2] \; f(m_{\nu_j}^2 / m_{\nu_2}^2), \tag{5.3}$$
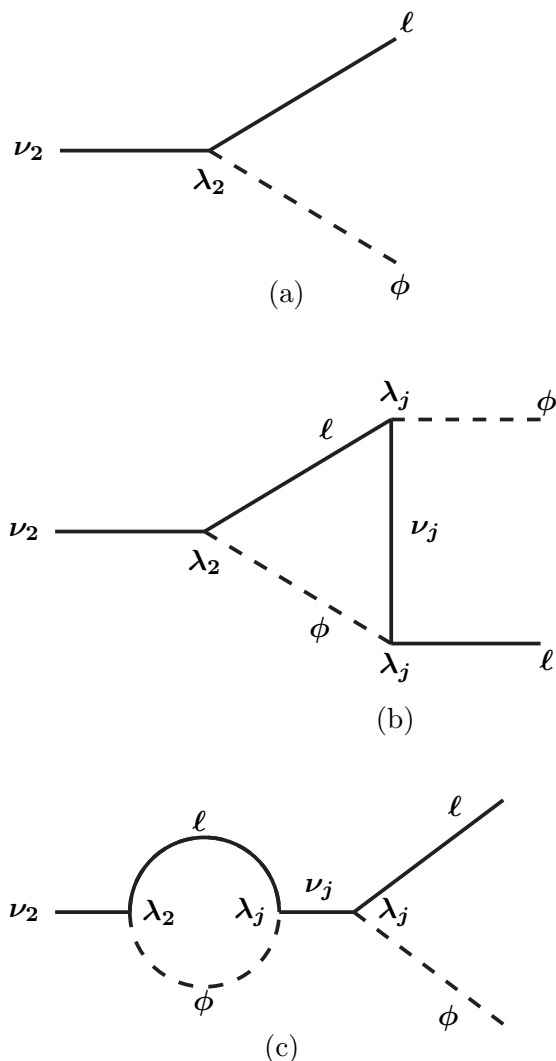
Figure 3: Diagrams for the CP-asymmetry decays, at tree (a) and loop (b)-(c) level, of the second neutrino eigenstate ν_2 into the charged lepton and the charged Higgs boson.

where f is the loop factor [24], given by

$$f(x) = \sqrt{x}\left[1 - (1+x)\ln\frac{1+x}{x} + \frac{1}{1-x}\right], \tag{5.4}$$

in the case of a hierarchical neutrino spectrum. Here ν_2, the next-to-lightest neutrino, is the external (i.e. decaying) mode and ν_j the exchanged eigenstate (see figure 3). The amount of CP asymmetry generated can be magnified by the loop function and/or the Yukawa couplings. However, due to the large hierarchy between the lightest first state ν_1 and the next-to-lightest state ν_2, much enhancement from the Yukawa couplings cannot be expected, as we will discuss more precisely in the next subsection. The other enhancement may then come from the loop function via resonances. The discussion on the characteristics of the generated KK neutrino spectrum in the previous section clearly shows that we

are indeed often in this favorable resonance situation ($\Leftrightarrow \varepsilon \sim 1/2R$), since the neutrino eigenstates are almost pairwise degenerate over a wide range of the obtained parameter space. In this situation, the self energy contribution to the loop factor, which is the last term in eq. (5.4), is modified to [25]

$$f^{\text{res}}(m_{\nu_j}^2/m_{\nu_2}^2) = \frac{(m_{\nu_2}^2 - m_{\nu_j}^2)m_{\nu_2}\Gamma_{\nu_j}}{(m_{\nu_2}^2 - m_{\nu_j}^2)^2 + m_{\nu_2}^2\Gamma_{\nu_j}^2}, \tag{5.5}$$

where $\Gamma_{\nu_j} \simeq K_{jj}m_{\nu_j}/8\pi$ is the decay width of state ν_j.

5.2 Results and discussion

As before, we have scanned the total parameter space with the additional constraints mentioned above concerning leptogenesis, concentrating on the scenario with $n_g = 1$. We checked that within the previously obtained parameter space, the first test to ensure leptogenesis (represented by eq. (5.1)), is satisfied.

As shows eq. (5.3), the source of a non-zero CP asymmetry is the difference of phases of the Yukawa couplings. Since we are working with one generation of neutrinos, there is a single (common) phase associated with the Yukawa couplings λ' (the one associated with the SM neutrino). However, the final (modified) Yukawa couplings λ_j may still be different for different neutrino mass eigenstates ν_j, as V itself is a complex matrix. The resulting difference of phases, generated in the process of diagonalization of the mass matrix as described above, leads to a non-zero value of $\text{Im}[\lambda^\dagger\lambda]_{2j}^2$.

However, although non-zero, $\text{Im}[\lambda^\dagger\lambda]_{2j}^2$ is small due to the large hierarchies among the elements of the mass matrix, which in turn is essential to generate a realistic neutrino spectrum with small active-sterile mixing angles. We find numerically that the resonance enhancement in ϵ_{CP} from the loop function cannot compensate this suppression from the Yukawa couplings in order to ultimately generate a sufficient CP asymmetry for a successful leptogenesis: $\epsilon_{\text{CP}} > 10^{-8}$ [26] (assuming the usual orders of magnitude for the washout factors). Thus, although the pairwise quasi-degenerate KK neutrino spectrum for one-generation of SM neutrino is favorable for yielding a good value of ϵ_{CP}, but the suppression of the same from the Yukawa couplings turns out to be too dominant.

A possible alternative for a potentially successful leptogenesis could be to probe a more realistic model with three generations of SM neutrinos on the 3-brane, with three extra right handed neutrinos in the bulk. The mass matrix in eq. (2.16) in that case would have a similar structure with h_1 and h_2 replaced by 3×3 matrices in the flavor space. The masses of the nth neutrino states would remain approximately the same for each generation and the differences in the masses of the three light neutrinos would only come from the Yukawa couplings. This initial difference in the Yukawa couplings could be enhanced further during the process of diagonalization of the mass matrix. This is in contrast to the one-generation case where we had to start from a single Yukawa coupling, and the process of matrix diagonalization was the *only* source to create a difference in the final Yukawa coupling phases. Therefore, we could expect a much larger contribution to the CP asymmetry from the $\text{Im}[\lambda^\dagger\lambda]$ factor, compared to the one-generation case.

6. Conclusions

In this work, we have considered an L-breaking extra dimensional scenario, with the SM particles localized on a 3-brane and an additional right handed neutrino propagating along an extra dimension, providing new mechanisms for suppressing the neutrino mass. We probed the possibilities of this model to account simultaneously for a realistic neutrino mass spectrum and a sufficient lepton asymmetry (from the decays of the KK neutrinos). We considered two variants of the above scenario.

The first is when this extra dimension is the only available one ($n_g = 1$). Then there exist fundamental parameters which give rise to a neutrino spectrum respecting the experimental constraints, on neutrino masses and active-sterile neutrino mixings (like the SNO bounds). We have found that in this framework, the neutrino mass suppression originates dominantly from the ED see-saw mechanism and partially from the ED wave function overlap effect. In fact, the SNO data limit the effect of the wave function overlap but a sufficiently small neutrino mass can be generated thanks to the ED see-saw mechanism. For certain parameter domains, the spectra in this scenario consists of pairwise quasi-degenerate heavy (KK) neutrinos, together with a light neutrino. We observed that under the constraints on neutrino masses that we have imposed, a "brane shifted" framework of the extra dimensional model is not essential for generating a realistic mass spectra. Also, the inverse radius of compactification has to be pushed to $\sim 400 \, \text{GeV}$ (or more). This in turn results in very tiny phase differences among the modified Yukawa couplings (after the process of mass matrix diagonalization) so that it is not possible to create a CP asymmetry large enough to insure a successful leptogenesis. Indeed, such a tininess is not sufficiently compensated by the enhancement from the resonant loop function (related to the decay of the KK neutrinos) which is due to the pairwise quasi-degeneracy property of the spectrum.

In the second variant where gravity can propagate in more extra dimensions ($n_g > 1$), the fundamental gravity scale is reduced down to the TeV scale so that the gauge hierarchy problem is addressed through the ADD approach. We have found that in this framework, the neutrino mass suppression originates also primarily from the ED see-saw mechanism and the compactification scale is of an order of magnitude much larger than the fundamental gravity scale.

A. Approximate analytical estimation of KK-neutrino masses

Here we intend to show analytically that when the eigenvalues ($\mathcal{E}_n$ with $n \geq 2$) of mass

matrix (2.16) are taken equal to the diagonal elements, their correction are much smaller than the eigenvalues themselves. For that purpose, let us start with the 4×4 form of eq. (2.16),

$$\mathcal{M} = \begin{pmatrix} 0 & m^{(0)} & m^{(-1)} & m^{(1)} \\ m^{(0)} & \varepsilon & 0 & 0 \\ m^{(-1)} & 0 & \varepsilon - \frac{1}{R} & 0 \\ m^{(1)} & 0 & 0 & \varepsilon + \frac{1}{R} \end{pmatrix}. \tag{A.1}$$

All the obtained sets of parameters (see discussion on figure 1) lead to a significant hierarchy in m/ε (case (1) of eq. (3.4)). Hence the $m^{(n)}$ values are systematically much smaller than ε, from eq. (2.17), namely: $m^{(n)} \ll \varepsilon$. We can thus rewrite matrix in eq. (A.1) as $\mathcal{M} = \mathcal{M}_D + \mathcal{M}_I$, where $\mathcal{M}_D$ is the diagonal matrix constructed from $\mathcal{M}$, and $\mathcal{M}_I$ is a perturbation matrix with no diagonal entries given as,

$$\mathcal{M}_I = \begin{pmatrix} 0 & m^{(0)} & m^{(-1)} & m^{(1)} \\ m^{(0)} & 0 & 0 & 0 \\ m^{(-1)} & 0 & 0 & 0 \\ m^{(1)} & 0 & 0 & 0 \end{pmatrix}. \tag{A.2}$$

The zeroth order eigenvalues $\mathcal{E}_i$ of $\mathcal{M}$ can be written as, $\mathcal{E}_1 = 0, \mathcal{E}_2 = \varepsilon, \mathcal{E}_3 = \varepsilon - 1/R, \mathcal{E}_4 = \varepsilon + 1/R$. Since $\mathcal{M}_I$ has all diagonal elements equal to zero, the lowest order correction to the eigenvalues reads as,

$$\delta\mathcal{E}_n = \sum_{n' \neq n} \frac{(<\psi_n|\mathcal{M}_I|\psi_{n'}>)^2}{\mathcal{E}_n - \mathcal{E}_{n'}}, \tag{A.3}$$

where the ψ's are the zeroth order eigenvectors (considering the non-degenerate case). Thus, for our case we have from the above relations,

$$\delta\mathcal{E}_1 = \frac{(m^{(0)})^2}{-\varepsilon} + \frac{(m^{(1)})^2}{-\varepsilon - 1/R} + \frac{(m^{(-1)})^2}{-\varepsilon + 1/R} \tag{A.4}$$

$$\delta\mathcal{E}_n = (m^{(n)})^2/\mathcal{E}_n, \tag{A.5}$$

where $m^{(n)}$ is given by eq. (2.17) and $n > 1$. Since $m \ll \varepsilon$, the first two of the above relations lead to

$$\delta\mathcal{E}_1 \leq m^2 \left[\frac{2\varepsilon}{(1/R)^2 - (\varepsilon)^2} - \frac{1}{\varepsilon} \right], \tag{A.6}$$

$$\delta\mathcal{E}_2 \leq m^2/\varepsilon. \tag{A.7}$$

These inequalities can immediately be generalized to the $N \times N$ mass-matrix as,

$$\delta\mathcal{E}_1 \leq m^2 \left[\sum_{n=1}^{(N-1)/2} \frac{2\varepsilon}{(n/R)^2 - (\varepsilon)^2} - \frac{1}{\varepsilon} \right], \tag{A.8}$$

$$\delta\mathcal{E}_2 \leq m^2/\varepsilon. \tag{A.9}$$

For the maximum value of ε $(= 1/2R)$, the first term on the right hand side of eq. (A.8) is $\sim 2R$ for large N, so that $\delta\mathcal{E}_1 \sim 0$. For smaller values of ε, $\delta\mathcal{E}_1$ would be dominated by the

second term on right hand side. For all the other eigenvalues with $n > 2$, one can derive $\mathcal{E}_n \geq \varepsilon$. Thus, in general the numerical value of $\delta\mathcal{E}_n$ for *any* n verifies

$$|\delta\mathcal{E}_n| \leq m^2/\varepsilon. \tag{A.10}$$

So let us study the m^2/ε value. For $R^{-1} \lesssim 10^8\,\mathrm{GeV}$, $\varepsilon \gtrsim 200\,\mathrm{GeV}$ (see figure 1) and $m \lesssim 10^{-1}\,\mathrm{GeV}$ (c.f. eq. (2.18) and eq. (2.5) in case of the scenario with $n_g = 1$). Hence, one has $m^2/\varepsilon \lesssim 10^{-5}\,\mathrm{GeV}$. Now for $R^{-1} \gtrsim 10^8\,\mathrm{GeV}$, $\varepsilon \sim R^{-1}/2$ and $m \sim 10^{-4}\,(R^{-1}/1\mathrm{GeV})^{1/3}\,\mathrm{GeV}$, so that $m^2/\varepsilon \sim 10^{-8}\,(R^{-1}/1\mathrm{GeV})^{-1/3}\,\mathrm{GeV}$ and thus $m^2/\varepsilon \lesssim 10^{-11}\,\mathrm{GeV}$. Therefore, from eq. (A.10), we conclude that one has systematically $|\delta\mathcal{E}_n| \ll \mathcal{E}_n$ for each $n \geq 2$, since $\mathcal{E}_{n \geq 2} \gtrsim \varepsilon \gtrsim 200\,\mathrm{GeV}$. A similar analysis could be performed for the case of the degenerate mass spectra (then we would have to start from the corresponding relation of eq. (A.3) for degenerate eigenvalues).

References

[1] N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, *The hierarchy problem and new dimensions at a millimeter*, *Phys. Lett.* **B 429** (1998) 263 [hep-ph/9803315];
I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, *New dimensions at a millimeter to a Fermi and superstrings at a TeV*, *Phys. Lett.* **B 436** (1998) 257 [hep-ph/9804398];
K.R. Dienes, E. Dudas and T. Gherghetta, *Extra spacetime dimensions and unification*, *Phys. Lett.* **B 436** (1998) 55 [hep-ph/9803466]; *Grand unification at intermediate mass scales through extra dimensions*, *Nucl. Phys.* **B 537** (1999) 47 [hep-ph/9806292].

[2] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [hep-ph/9905221].

[3] I. Antoniadis, *A possible new dimension at a few TeV*, *Phys. Lett.* **B 246** (1990) 377;
J.D. Lykken, *Weak scale superstrings*, *Phys. Rev.* **D 54** (1996) 3693 [hep-th/9603133];
P. Hořava and E. Witten, *Heterotic and type-I string dynamics from eleven dimensions*, *Nucl. Phys.* **B 460** (1996) 506 [hep-th/9510209]; *Eleven-dimensional supergravity on a manifold with boundary*, *Nucl. Phys.* **B 475** (1996) 94 [hep-th/9603142].

[4] K.R. Dienes, E. Dudas and T. Gherghetta, *Light neutrinos without heavy mass scales: a higher-dimensional seesaw mechanism*, *Nucl. Phys.* **B 557** (1999) 25 [hep-ph/9811428].

[5] N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali and J. March-Russell, *Neutrino masses from large extra dimensions*, *Phys. Rev.* **D 65** (2002) 024032 [hep-ph/9811448].

[6] A. Pilaftsis, *Leptogenesis in theories with large extra dimensions*, *Phys. Rev.* **D 60** (1999) 105023 [hep-ph/9906265].

[7] A. Ioannisian and A. Pilaftsis, *Cumulative non-decoupling effects of Kaluza-Klein neutrinos in electroweak processes*, *Phys. Rev.* **D 62** (2000) 066001 [hep-ph/9907522].

[8] M. Maltoni et al., *Status of global fits to the neutrino oscillations*, *New. J. Phys.* **6** (2004) 122.

[9] For example see: G.R. Dvali and A.Y. Smirnov, *Probing large extra dimensions with neutrinos*, *Nucl. Phys.* **B 563** (1999) 63 [hep-ph/9904211];
R.N. Mohapatra, S. Nandi and A. Perez-Lorenzana, *Neutrino masses and oscillations in models with large extra dimensions*, *Phys. Lett.* **B 466** (1999) 115 [hep-ph/9907520];

A. Lukas, P. Ramond, A. Romanino and G.G. Ross, *Solar neutrino oscillation from large extra dimensions*, *Phys. Lett.* **B 495** (2000) 136 [`hep-ph/0008049`];

R. Barbieri, P. Creminelli and A. Strumia, *Neutrino oscillations from large extra dimensions*, *Nucl. Phys.* **B 585** (2000) 28 [`hep-ph/0002199`];

R.N. Mohapatra and A. Perez-Lorenzana, *Three flavour neutrino oscillations in models with large extra dimensions*, *Nucl. Phys.* **B 593** (2001) 451 [`hep-ph/0006278`];

A. Lukas, P. Ramond, A. Romanino and G.G. Ross, *Neutrino masses and mixing in brane-world theories*, *JHEP* **04** (2001) 010 [`hep-ph/0011295`];

K.R. Dienes and I. Sarcevic, *Neutrino flavor oscillations without flavor mixing angles*, *Phys. Lett.* **B 500** (2001) 133 [`hep-ph/0008144`];

N. Cosme et al., *Neutrino suppression and extra dimensions: a minimal model*, *Phys. Rev.* **D 63** (2001) 113018 [`hep-ph/0010192`];

D.O. Caldwell, R.N. Mohapatra and S.J. Yellin, *Large extra dimensions, sterile neutrinos and solar neutrino data*, *Phys. Rev. Lett.* **87** (2001) 041601 [`hep-ph/0010353`];

J. Maalampi, V. Sipilainen and I. Vilja, *Neutrinos confronting large extra dimensions*, *Phys. Lett.* **B 512** (2001) 91 [`hep-ph/0103312`];

C.S. Lam and J.N. Ng, *Neutrino oscillations via the bulk*, *Phys. Rev.* **D 64** (2001) 113006 [`hep-ph/0104129`];

A.S. Dighe and A.S. Joshipura, *Neutrino anomalies and large extra dimensions*, *Phys. Rev.* **D 64** (2001) 073012 [`hep-ph/0105288`];

A. Nicolaidis and D.T. Papadamou, *Manifestations of extra dimensions in a neutrino telescope*, *Phys. Rev.* **D 66** (2002) 013005 [`hep-ph/0109048`];

J.M. Frere, M.V. Libanov and S.V. Troitsky, *Neutrino masses with a single generation in the bulk*, *JHEP* **11** (2001) 025 [`hep-ph/0110045`];

J.L. Hewett, P. Roy and S. Roy, *Higher dimensional models of light majorana neutrinos confronted by data*, *Phys. Rev.* **D 70** (2004) 051903 [`hep-ph/0404174`];

N. Arkani-Hamed and S. Dimopoulos, *New origin for approximate symmetries from distant breaking in extra dimensions*, *Phys. Rev.* **D 65** (2002) 052003 [`hep-ph/9811353`];

T. Gherghetta, *Dirac neutrino masses with planck scale lepton number violation*, *Phys. Rev. Lett.* **92** (2004) 161601 [`hep-ph/0312392`];

N. Haba, *5D seesaw, flavor structure and mass textures*, *JHEP* **05** (2006) 030 [`hep-ph/0603119`];

M.-T. Eisele and N. Haba, *Family symmetry and single right-handed neutrino dominance in five dimensions*, *Phys. Rev.* **D 74** (2006) 073007 [`hep-ph/0603158`].

[10] A. Lukas, P. Ramond, A. Romanino and G.G. Ross, *Neutrino masses and mixing in brane-world theories*, *JHEP* **04** (2001) 010 [`hep-ph/0011295`].

[11] G. Bhattacharyya, H.V. Klapdor-Kleingrothaus, H. Pas and A. Pilaftsis, *Neutrinoless double beta decay from singlet neutrinos in extra dimensions*, *Phys. Rev.* **D 67** (2003) 113001 [`hep-ph/0212169`].

[12] H. Davoudiasl, P. Langacker and M. Perelstein, *Constraints on large extra dimensions from neutrino oscillation experiments*, *Phys. Rev.* **D 65** (2002) 105015 [`hep-ph/0201128`].

[13] M. Fukugita and T. Yanagida, *Baryogenesis without grand unification*, *Phys. Lett.* **B 174** (1986) 45.

[14] For example, see E.G. Gimon and J. Polchinski, *Consistency conditions for orientifolds and D-manifolds*, *Phys. Rev.* **D 54** (1996) 1667 [`hep-th/9601038`].

[15] G. Moreau, *Realistic neutrino masses from multi-brane extensions of the randall-sundrum model?*, *Eur. Phys. J.* **C 40** (2005) 539 [hep-ph/0407177]; *Realistic neutrino masses from multi-brane extensions of the randall-sundrum model?*, *Eur. Phys. J.* **C 40** (2005) 539 [hep-ph/0407177].

[16] M. Cirelli, G. Marandella, A. Strumia and F. Vissani, *Probing oscillations into sterile neutrinos with cosmology, astrophysics and experiments*, *Nucl. Phys.* **B 708** (2005) 215 [hep-ph/0403158].

[17] P. Aliani, V. Antonelli, M. Picariello and E. Torrente-Lujan, *The neutrino mass matrix after kamland and SNO salt enhanced results*, hep-ph/0309156.

[18] A. Bandyopadhyay, S. Choubey, S. Goswami, S.T. Petcov and D.P. Roy, *Constraints on neutrino oscillation parameters from the SNO salt phase data*, *Phys. Lett.* **B 583** (2004) 134 [hep-ph/0309174].

[19] P. Minkowski, $mu \to e$ *gamma at a rate of one out of 1-billion muon decays?*, *Phys. Lett.* **B 67** (1977) 421;
M. Gell-Mann, P. Ramond and R. Slansky, *Proceedings of the supergravity stony brook workshop*, P. Van Nieuwenhuizen and D. Freedman eds., New York (1979);
T. Yanagida, *Proceedings of the workshop on unified theories and baryon number in the universe*, A. Sawada and A. Sugamoto eds., Tsukuba, Japan (1979);
R.N. Mohapatra and G. Senjanovic, *Neutrino mass and spontaneous parity nonconservation*, *Phys. Rev. Lett.* **44** (1980) 912.

[20] SNO collaboration, Q.R. Ahmad et al., *Measurement of the charged current interactions produced by b-8 solar neutrinos at the sudbury neutrino observatory*, *Phys. Rev. Lett.* **87** (2001) 071301 [nucl-ex/0106015].

[21] SUPER-KAMIOKANDE collaboration, S. Fukuda et al., *Solar B-8 and HEP neutrino measurements from 1258 days of super-kamiokande data*, *Phys. Rev. Lett.* **86** (2001) 5651 [hep-ex/0103032].

[22] T.-K. Kuo and J.T. Pantaleone, *Neutrino oscillations in matter*, *Rev. Mod. Phys.* **61** (1989) 937.

[23] See, e.g. T.M. Ito and G. Prezeau, *Neutrino mass constraints on beta decay*, *Phys. Rev. Lett.* **94** (2005) 161802 [hep-ph/0410254].

[24] L. Covi, E. Roulet and F. Vissani, *CP-violating decays in leptogenesis scenarios*, *Phys. Lett.* **B 384** (1996) 169 [hep-ph/9605319].

[25] A. Pilaftsis and T.E.J. Underwood, *Resonant leptogenesis*, *Nucl. Phys.* **B 692** (2004) 303 [hep-ph/0309342].

[26] G.F. Giudice, A. Notari, M. Raidal, A. Riotto and A. Strumia, *Towards a complete theory of thermal leptogenesis in the SM and MSSM*, *Nucl. Phys.* **B 685** (2004) 89 [hep-ph/0310123];
W. Buchmuller, P. Di Bari and M. Plumacher, *Leptogenesis for pedestrians*, *Ann. Phys. (NY)* **315** (2005) 305 [hep-ph/0401240].

[27] V.A. Rubakov and M.E. Shaposhnikov, *Do we live inside a domain wall?*, *Phys. Lett.* **B 125** (1983) 136.

[28] N. Arkani-Hamed and M. Schmaltz, *Hierarchies without symmetries from extra dimensions*, *Phys. Rev.* **D 61** (2000) 033005 [hep-ph/9903417];

E.A. Mirabelli and M. Schmaltz, *Yukawa hierarchies from split fermions in extra dimensions*, *Phys. Rev.* **D 61** (2000) 113011 [`hep-ph/9912265`];

G.R. Dvali and M.A. Shifman, *Families as neighbors in extra dimension*, *Phys. Lett.* **B 475** (2000) 295 [`hep-ph/0001072`];

G.C. Branco, A. de Gouvea and M.N. Rebelo, *Split fermions in extra dimensions and CP-violation*, *Phys. Lett.* **B 506** (2001) 115 [`hep-ph/0012289`];

M. Raidal and A. Strumia, *Predictions of the most minimal see-saw model*, *Phys. Lett.* **B 553** (2003) 72 [`hep-ph/0210021`].

[29] H.V. Klapdor-Kleingrothaus and U. Sarkar, *Majorana neutrinos with split fermions in extra dimensions*, *Phys. Lett.* **B 541** (2002) 332 [`hep-ph/0201226`].